Improved Distributed Coverage Control for Robotic Visual Sensor Network under Limited Energy Storage

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ABSTRACT

A robotic sensor network is advantageous in performing a coverage task compared to the static sensor network due to its ability to self-deploy and self-reconfigure. However, since the sensor has a limited sensing range, when mobile sensors are initially deployed, sensors located far away from the region of interest may not be able to self-deploy themselves, i.e. participate in the coverage task. This results in a degradation of coverage performance by the robotic network. Furthermore, since in reality the mobile sensors have only a limited energy storage, the movement of the sensors have to be as efficient as possible. This article proposes a novel distributed algorithm in order to improve the coverage performance by the robotic visual sensor network by guaranteeing the participation of all sensors in the coverage task and considering the energy consumption of the sensors in the motion planning. The algorithm is a combination of the standard gradient-based coverage algorithm and a leader-following algorithm and is designed to maximize the joint detection probabilities of the events in the region of interest. In addition, the standard coverage control law is further modified in order to take into account the energy consumption of the sensors. The results are validated through numerical simulations.

Keywords: coverage control, mobile sensor network, distributed algorithm.

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1 Introduction

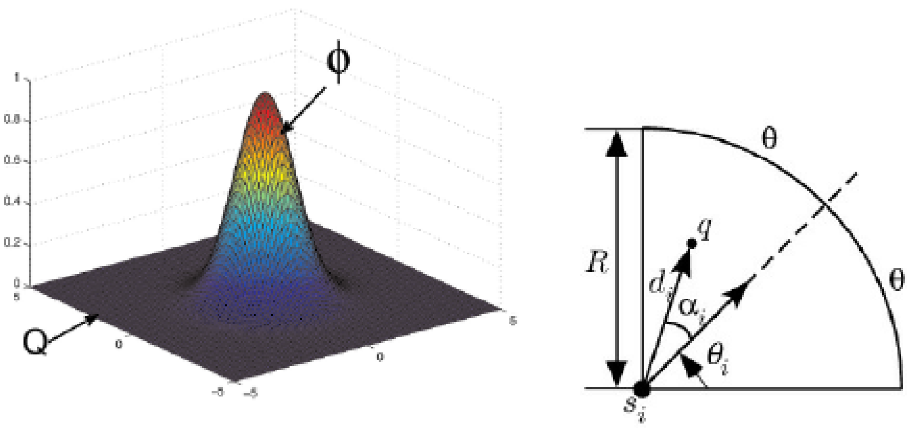
Stimulated by technological advances and development of relatively inexpensive communication, computation, and sensing devices, the interest in the research area of cooperative control of multi-agent systems has majorly increased over the last decades. In multi-agent systems, each individual is assumed to have abilities to sense its immediate environment, communicate with others, process information gathered and take a local action based on the information gathered. Key aspects of designing the control laws for motion coordination are that each agent has to behave without any leader (leader-less) and based on its local information (i.e. the control law is distributed) due to limited communication and sensing capabilities. This is also inspired by the behaviour of biological systems such as school of fish, flocking of birds and animal herds that exist in nature.

In this paper our focus is on deployment of a mobile sensing network of vehicles equipped with sensors to sample the environment. Such problem is called a coverage control problem. Coverage control has a closed relation with sensor networks. A sensor network consists of a collection of sensing devices that can coordinate their actions through wireless communication and aim at performing tasks such as collecting data over a region of interest. Sensor networks usually consist of stationary sensor nodes. The deployment of a static network is often either human monitored or random, depending on the environment. It can be predetermined when the environment is sufficiently known, in which case, sensors can be strategically hand placed e.g. the art gallery problem where the goal is to determine position and minimal number of cameras in a building or gallery so that every point in the building is observed by at least one camera. However, if the environment is unknown, hostile or even dangerous for humans, the deployment cannot be determined *a priori* in which case sensors may be air-dropped from an aircraft, generally resulting in a random configuration. Sometimes, deploying such a stationary sensor network and maintaining its sensing coverage could be a difficult task. For example, imagine deploying a stationary sensor network over a region of interest. Even though advanced tools like airplanes are available to make the deployment safer and easier, various factors such as winds and obstacles are very likely to introduce coverage holes regardless of how many sensor nodes are dropped. Even if a perfect coverage can be achieved initially, events such as sensor failures will certainly degrade coverage performance as time evolves.

Due to the problem of stationary sensor networks, there is an urgent need for sensor nodes to be equipped with mobility for instance by installing sensors on autonomous vehicles or robots. Mobility brings some improvements to the performance of sensor networks. First, the sensor nodes are able to autonomously re-deploy themselves that maximizes the coverage of the environment. Second, by adapting their configuration, the network will remain robust to environment changes due to sensors departures, arrivals or malfunction and also communication links' failures. Moreover, mobile sensors can undertake challenging tasks that can be too dangerous for human intervention such as search and recovery operations, manipulation in hazardous environments, surveillance, wild fire detection and also environmental monitoring for pollution detection and estimation (Leonard, Paley, Lekien, Sepulchre, Fratantoni and Davis, 2007), (Paley, Zhang and Leonard, 2008).

In general there are three main approaches to solve the coverage control problem using mobile sensor networks namely geometric, probabilistic, and potential field approach. The geometric strategy is based on the Voronoi partition and the continuous version of Lloyd algorithm, see e.g. (Cortes, Martinez, Karatas and Bullo, 2004). Briefly speaking, the agents partition the given region into subregions given by Voronoi partitions and move towards the centroid of its subregion and adjust its sensing radius until all the area is covered. The advantage of the Voronoi approach is that the control law is distributed by its nature. Moreover, a lot of variations on the coverage control problem can be addressed and solved in a similar fashion by associating different Voronoi partition. Pimenta et al. (Pimenta, Kumar, Mesquita and Pereira, 2008) used the so-called power diagram to deal with coverage control problem with heterogeneous robots, i.e., robots with different footprints. The probabilistic based strategy is introduced in (Li and Cassandras, 2005) where the authors consider a probabilistic network model and a density function to represent the frequency of random events taking place over the mission space. The authors develop an optimization problem that aims at maximizing coverage using sensors with limited ranges, while minimizing communication cost. A potential-field-based approach to deployment problem in an unknown environment is presented in (Howard, Mataric and Sukhatme, 2002). An algorithm based on similar approach is proposed in (Poduri and Sukhatme, 2004) that maximizes the area coverage of a network while satisfying the constraint that every node has at least K neighbors. Moreover, coverage control problem based on receding horizon control is considered in (Ahmadzadeh, Jadbabaie and Kumar, 2007). Coverage control problem considering a more realistic sensor model is considered in (Cortes, Martinez and Bullo, 2005) by introducing "limited-range interactions" of the sensors, i.e the sensing range of the sensor is restricted to a bounded region. Furthermore, the coverage algorithm for robotic visual sensor networks is proposed in (Gusrialdi, Hatanaka and Fujita, 2008; Hatanaka, Ibuki, Gusrialdi and Fujita, 2009) where the sensor has a limited view angle and its sensing performance also depends on its attitude.

When the mobile sensors are initially deployed in the unknown environment, some sensors may be located far away from the region of interest. Moreover, due to the limited sensing range of the sensors, those sensors may not be able to participate in the coverage task. All of the works mentioned above have not considered this issue. Thus, the first contribution of this article is the development of a novel algorithm for coverage control of mobile sensor networks that guarantees the participation of all sensors in the coverage task, even if some sensors do not sense any event in the initial deployment. The idea is to combine the standard coverage algorithm with the leader-following algorithm where the leader(s), i.e., sensor(s) who has sufficient information about the environment, is selected using a voting mechanism. The sensors acting as leaders will then guide the sensors which do not have information on the environment until they gain sufficient information. Furthermore, in reality, the mobile sensors have limited amount of power to move, thus it is required to incorporate the current energy state for the deployment of the mobile sensors. The authors in (Kwok and Martinez, 2010) considered power-aware coverage algorithms for mobile networks in order to balance the energy expenditure accross the network and make nodes with high power compensate for nodes with low power by incorporating partition defined by power metric. However, only isotropic sensor is considered. Thus, the second contribution of this article is the development of distributed coverage algo-



(a) Environment (b) Visual sensor model

Figure 1: (a) The region of interest and its density function $\phi(q)$. Regions with a high value of ϕ are regions of higher chances of finding a point of interest. In general, there are some regions in Q where $\phi(q) = 0$; (b) Visual sensor model.

rithms for robotic visual sensors incorporating the energy status of each sensor. Our work in this article is based on the coverage algorithm developed in (Li and Cassandras, 2005) since it can be generalized to non-convex environment (Gusrialdi and Zeng, 2011) in a straightforward manner.

This article is organized as follows: The problem formulation for the coverage problem is presented in Section 2. Power aware distributed coverage algorithm which at the same time guarantees the participation of all sensors in the coverage task is proposed in Section 3. The effectiveness of the proposed algorithm is validated through numerical simulations in Section 4. The short version of this article is appeared in (Dirza and Gusrialdi, 2011).

2 Problem Formulation

In this article we adopt some of the notation and setting described in (Li and Cassandras, 2005) and in addition introduce a new term called *information value* of a sensor which will be presented in the following.

2.1 Region of Interest

Let Q be a polyhedron in $\mathbb{R}^2$ including its interior. The density function $\phi(q) : Q \to \mathbb{R}_+$ represents the probability that some event take place in Q. Regions with a large value of ϕ are regions of higher chances of finding a point of interest. The density function $\phi(q)$ satisfies $\phi(q) \geq 0$ for all $q \in Q$ and $\int_Q \phi(q) < \infty$. In general, the region of interest Q may be very large so that there exists some areas where the value of $\phi(q)$ approaches 0 as illustrated in Fig. 1(a).

2.2 Sensor Model

We consider a robotic network where each robot is equipped with limited-range omnidirectional communication and anisotropic sensing capabilities. Let s $= (s_1, ..., s_N)$ be the location of the N identical robots/sensors moving in the region Q. Let $\theta = (\theta_1, ..., \theta_N)$ be the orientation/attitude of the N sensors. The kinematic model of the sensors are given by

$$\begin{aligned} \dot{s_i} &= u_i, \\ \dot{\theta_i} &= v_i. \end{aligned} \tag{2.1}$$

When an event occurs at point q, it emits a signal and this signal is observed by sensor i at location s_i. The received signal strength, i.e. the sensing performance of the sensor is assumed to be decayed not only with the distance from the sensor but also with the orientation of the target to the sensor. We define this type of sensor as an *anisotropic* sensor model. In this article we consider a pin-hole type camera as a sensor due to its general versatility and is a typical example of an anisotropic sensor. The degradation of the sensor performance is represented by a monotonically decreasing differentiable function $p_i(q)$, which expresses the probability that sensor i detects the event occuring at q or indicates how poor the sensing performance is. Lower value of $p_i(q)$ means that point q is sensed poorly by sensor i and vice versa. Moreover, as a novelty in this article, the sensing range R_i is assumed to be influenced by embodied energy E_i. Formally, the visual sensor model as shown in Fig. 1(b) is given as follows:

Model 1. *Each sensor has a limited sensory domain Q_i with sensing range R_i and sensing direction θ_i. The sensing ability of each sensor declines along the radial distance and the radial angle from the sensor to the point to be sensed. Moreover, the sensing range R_i depends on the ratio between the current embodied energy E_i and the maximum energy capacity E_i^{max} of sensor i. We define the sensing range in the case of full capacity of embodied energy as R_i^{max}. Mathematically, the sensory domain of each sensor is given by*

$$Q_i = \{q \in Q : d_i \leq R_i(E_i) \cap |\alpha_i| \leq \Theta\} \tag{2.2}$$

where

$$\begin{aligned} d_i &= \|q - s_i\| \\ \alpha_i &= \cos^{-1} \frac{\Big(q - s_i\Big)\Big(\cos\theta_i, \sin\theta_i\Big)}{\Big\|q - s_i\Big\|} \\ \Theta &\in (0, \frac{\pi}{2}] \\ R_i(E_i) &= \frac{E_i}{E_i^{max}} R_i^{max}. \end{aligned}$$

In the remainder of the article, for the sake of simplicity it is assumed that $E_i^{\max} = \cdots = E_N^{\max} = E_{\max}$ and $R_i^{\max} = \cdots = R_N^{\max} = R_{\max}$ respectively. Moreover we make the following assumption on the sensing performance of the above sensor model.

Assumption 1.

$$p_i(q) = 0, \frac{\partial p_i(q)}{\partial d_i(q)} = 0, \frac{\partial p_i(q)}{\partial \alpha_i(q)} = 0 \text{ if } q \notin Q_i \tag{2.3}$$

The assumption tells us that the sensor i can only sense the point inside its region of sensing Q_i. An example of the sensing performance is given for example by

$$p_i(q) = \begin{cases} \dfrac{\Big(d_i - R_i(E_i)\Big)^2 \Big(\alpha_i - \Theta\Big)^2}{\Big(R_{max}\Theta\Big)^2} & \text{if } q \in Q_i \\ 0 & \text{otherwise.} \end{cases} \tag{2.4}$$

Next we define the amount of information sensed by each sensor, i.e. I_i given as follows

Definition 2.1. The information sensed by sensor i is defined as the total received signal strength inside its sensory domain Q_i. Mathematically, it is formulated as

$$I_i(q) = \int_{Q_i} \phi(q) p_i(q, s_i, \theta_i, E_i) dq. \tag{2.5}$$

Next we define the communication structure of the robotic sensor network by assuming that it is represented by the r-disk proximity graph defined below.

Definition 2.2. The pair (I, B_{disk}) is said to be an $r-$disk proximity graph if and only if the edge map $B_{disk} : Q^N \to I \times I$ satisfies

$$(i, j) \in B_{disk}(s) \iff i \neq j \text{ and } \|s_i - s_j\| \leq r.$$

2.3 Energy Consumption Model

In this subsection we describe the model for energy consumption of the mobile sensors. In this article, we use the analogy of multiplicatively weighted metric introduced in (Kwok and Martinez, 2010) and extend it to visual sensor model in order to analyze the energy content of each sensor E_i which means that the ownership of a free point q is changed over as a result of updating d_i and R_i. Here, it is assumed that each sensor has an "energy content" quantified by $E_i \in [0, E_{max}] \subseteq \Re$. Let k_e be the weigthing factor for the corresponding energy consumption characteristic. Moreover, let $E = (E_1, E_2, ..., E_N)$ be the set of embodied energy of all sensors.

Definition 2.3. The motion of sensor i both velocity $\dot{s}_i(t)$ and angle velocity $\dot{\theta}_i(t)$ consume energy. Mathematically, the energy consumption is modelled as follows

$$\dot{E}_i(t) = -k_e \beta_t \frac{E_i^2}{E_{max}^2} \left(\left\| \dot{s}_i(t) \right\|^2 + \left\| \dot{\theta}_i(t) \right\|^2 \right) \tag{2.6}$$

where $E_i(t) \geq 0$.

Note that since it is assumed that the capacity of the embodied energy E_i influences the sensing performance of sensor i as shown in (2.4), $R_i(E_i)$ will also be dynamically changed.

2.4 Optimal Coverage Formulation

The optimal coverage is achieved by deploying the sensors into the region of interest so that the probability of events are detected is maximized. In this paper, agents are assumed to make observations independently. When an event at q is observed by the sensors, the joint probability that this event is detected can be written as

$$P(q,s,\theta,E) = 1 - \prod_{i=1}^{N}[1 - P_i(q,s,\theta,E)]. \tag{2.7}$$

Then, the optimal coverage problem can be formulated as an optimization problem which maximizes the objective function defined as

$$F(q,s,\theta,E) = \int_Q \phi(q)P(q,s,\theta,E)dq, \tag{2.8}$$

which is the expected event detection probability by the sensors over the region of interest. In this case, the controllable variables are the state of s,θ and E. The goal of the coverage control problem is to derive the control laws u_i, v_i such that the objective function F in (2.8) is locally maximized.

3 Proposed Distributed Control Laws

In this section we propose a distributed control law which incorporates the energy status of the mobile sensor networks. In addition, we improve the coverage performance of the distributed control law by guaranteing the participation of the sensors in the coverage task.

3.1 Power-Aware Coverage Control Law

First we introduce a power-metric control law to achieve optimal coverage by taking into account the energy storage of the mobile sensor network. Since we have to consider the capacity of the embodied energy E_i which influences the sensing performance of sensor i as shown in (2.4) , then $R_i(E_i)$ is not constant anymore but dynamically changed as shown in (2.2) and can be written as $R_i(E_i) = \frac{E_i}{E_{max}}R_{max}$. In other words, the objective function will depend on how the status of sensor i (p_i, θ_i) relatively to q is and how "charged" it is.
Next we propose a new control law by incorporating the energy of the robotic visual sensor network given as

$$u_i(t) = u_i^{\text{ce}}(t) = \beta_t\left(2k_e A(Q_i)\frac{E_i^{m+2}}{E_{max}^4}\right)^{-1}\frac{\partial F}{\partial s_i(t)} \tag{3.1}$$

$$v_i(t) = v_i^{\text{ce}}(t) = \beta_t\left(2k_e A(Q_i)\frac{E_i^{m+2}}{E_{max}^4}\right)^{-1}\frac{\partial F}{\partial \theta_i(t)} \tag{3.2}$$

where β_t is a time difference that should be properly chosen, $A(Q_i) = \int_{q\in Q_i}\phi(q)dq$ and $m \geq 1$ is a constant value that influences the velocity of the mobile sensor. Smaller value of m may speed up the velocity of the sensor, but at the same time may reduce its performance. Furthermore, $\frac{\partial F}{\partial s_i(t)}, \frac{\partial F}{\partial \theta_i(t)}$ in (3.1), (3.2) respectively are given by (Gusrialdi et al., 2008)

$$\begin{aligned}\frac{\partial F}{\partial s_i} &= \int_Q \phi(q)\frac{\partial P(q,s,\theta,E)}{\partial s_i}dq = \int_Q \phi(q)\prod_{k\in N_i}[1-P_i(q)]\frac{\partial P_i}{\partial s_i}dq,\\ &= \int_{Q_i}\phi(q)\prod_{k\in N_i}[1-p_k(q)]\left(\frac{\partial p_i(q)}{\partial d_i}\frac{\partial d_i}{\partial s_i} + \frac{\partial p_i(q)}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial s_i}\right)dq,\end{aligned}$$

$$\begin{aligned}\frac{\partial F}{\partial \theta_i} &= \int_Q \phi(q)\frac{\partial P(q,s,\theta,E)}{\partial \theta_i}dq = \int_Q \phi(q)\prod_{k\in N_i}[1-P_i(q)]\frac{\partial P_i}{\partial \theta_i}dq.\\ &= \int_{Q_i}\phi(q)\prod_{k\in N_i}[1-p_k(q)]\left(\frac{\partial p_i(q)}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial \theta_i}\right)dq.\end{aligned}$$

where

$$\begin{aligned}\frac{\partial d_i}{\partial s_i} &= -\frac{q-s_i}{d_i(q)},\\ \frac{\partial \alpha_i}{\partial s_i} &= \frac{-b_i(q)}{d_i^2(q)\sqrt{d_i^2(q)-a_i^2(q)}}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}(q-s_i),\\ \frac{\partial \alpha_i}{\partial \theta_i} &= \frac{-b_i(q)}{\sqrt{d_i^2(q)-a_i^2(q)}},\\ a_i(q) &= [\cos\theta_i \sin\theta_i](q-s_i), b_i(q) = [-\sin\theta_i \cos\theta_i](q-s_i).\end{aligned}$$

Note that since the new control inputs are controlled by the actual condition of the agent such as energy status E_i and sensing area $A(Q_i)$, it is clear that there is a trade off between maintaining energy status and achieving coverage objective.

Theorem 3.1. *Consider mobile sensors whose kinematics given by (2.1) and the control input is given by (3.1), (3.2). Thus, under assumption 1, all agents will achieve the coverage goal optimally and embodied energy maintenance in the network is improved.*

Proof. The motion of each sensor can be written as $\dot{p}_i = u_i^{\mathsf{ce}}/\beta_t$ and $\dot{\theta}_i = \upsilon_i^{\mathsf{ce}}/\beta_t$. Let us consider the energy dynamics of the sensors defined in (2.6). It is clear that the chosen energy dynamics (2.6) results in an asymptotical decrease of E_i given by

$$E_i(t) = \frac{E_{max}^2}{1+E_{max}\int_0^t \frac{\partial F}{\partial p_i(s)}^2 ds} + \frac{E_{max}^2}{1+E_{max}\int_0^t \frac{\partial F}{\partial \theta_i(s)}^2 ds}, \tag{3.3}$$

Consider vector field $X = (X_1, ...X_N)$, where $X_i = (X_{p_i}, X_{\theta_i}, X_{E_i})$ for $i \in (1, ..., N)$.
The Lie derivative of F with respect to X can be computed as

$$\sum_{i=1}^{N}\left(X_{p_i}\frac{\partial F}{\partial p_i(t)} + X_{\theta_i}\frac{\partial F}{\partial \theta_i(t)} + X_{E_i}\frac{\partial F}{\partial E_i(t)}\right), \tag{3.4}$$

where Q_i is a function of p_i and θ_i. In order to guarantee that the control inputs (3.1), (3.2) drive sensor i to achieve the coverage goal and improve the embodied energy maintenance, we need to prove that $\mathcal{L}_X F \geq 0$. Let define vector field X by (3.1), (3.2) and (2.6) respectively. The substitution leads to :

$$\begin{aligned}&\sum_{i=1}^{N}\left(X_{p_i}\frac{\partial F}{\partial p_i(t)} + X_{\theta_i}\frac{\partial F}{\partial \theta_i(t)} + X_{E_i}\frac{\partial F}{\partial E_i(t)}\right)\\ &= \sum_{i=1}^{N}\left(2\beta_t k_e A(Q_i)\frac{E_i^{m+2}}{E_{max}^4} - \beta_t k_e \frac{E_i^2}{E_{max}^2}\frac{\partial F}{\partial E_i(t)}\right)\\ &= \frac{\beta_t k_e}{E_{max}^2}\sum_{i=1}^{N}E_i^2\left(2A(Q_i)\frac{E_i^m}{E_{max}^2} - \frac{\partial F}{\partial E_i(t)}\right) \geq 0.\end{aligned}$$

Next, we show that $2A(Q_i)\frac{E_i^m}{E_{max}^2} \geq \frac{\partial F}{\partial E_i(t)}$ as follows :

$$\frac{\partial F}{\partial E_i} = \int_{q\in Q_i} \phi(q) \prod_{k\in N_i} [1-P_k(q)] 2 \left(\frac{(\alpha_i - \Theta)}{R_{max}\Theta}\right)^2$$
$$(d_i - R_i(E_i)) \left(\frac{-R_{max}}{E_{max}}\right) dq.$$

Note that $k \in N_i$ implies that agent i can only sense its neighbors. Since $\prod_{k\in N_i}[1-P_k(q)] \leq 1$ and $\left(\frac{(\alpha_i-\Theta)}{\Theta}\right)^2 \leq 1$, then we have

$$2\int_{q\in Q_i} \phi(q) \left(\frac{E_i}{E_{max}^2} - \frac{d_i R_{max}}{E_{max}}\right) dq \geq \frac{\partial F}{\partial E_i}.$$

Furthermore, since

$$\int_{q\in Q_i} \phi(q) \left(\frac{E_i}{E_{max}^2}\right) dq \geq \int_{q\in Q_i} \phi(q) \left(\frac{E_i}{E_{max}^2} - \frac{d_i R_{max}}{E_{max}}\right) dq,$$

we have

$$\left(\frac{2E_i^m}{E_{max}^2}\right) \int_{q\in Q_i} \phi(q) \geq \frac{\partial F}{\partial E_i}.$$

Thus, it can be concluded that $\frac{dF}{dt} \geq 0$. Since $m \geq 1$, i.e. the velocity of the sensors is limited, it guarantees that the objective function is maximized. The set of points $(\dot{p_i}, \dot{\theta}_i, \dot{E}_i, E_i)$ where $\mathcal{L}_X F = 0$ is given by

$$(\dot{p_i} = 0 \text{ and } \dot{\theta}_i = 0 \text{ and } \dot{E}_i = 0) \text{ or } (E_i = 0).$$

Since Q is a compact set, by LaSalle's invariance principle, the sensors will approach the largest invariant set contained in $\mathcal{L}_X F = 0$. This completes the proof. □

Remark 3.1. The above continuous calculation should be discretized in order to make it computable by each sensor by using similar method in (Li and Cassandras, 2005).

When the mobile sensors are initially deployed, e.g. air-dropped from an aircraft, some sensors may be located far away from the region of interest. Morever, since the control laws (3.1),(3.2) are based on a gradient-based approach and due to the limited sensing range of the sensor, there exists a condition at the initial deployment where the information gained by some agents are zero, i.e. $I_i(q, s_i, \theta_i) = 0$ which results in that the control input of the sensor $u_i^{\text{ce}}(k) = 0$, $v_i^{\text{ce}}(k) = 0$, i.e. the sensor could not participate in the coverage task which results in a coverage performance degredation. Formally, we call such a sensor as an *isolated* sensor defined as

Definition 3.1. Sensor i is called an isolated sensor if it collects no information so that it has no ability to move, i.e. $I_i = 0$.

Therefore, next we extend our power-aware control laws (3.1),(3.2) such that it is guaranteed that no isolated sensors exist in the final configuration of the mobile sensors, i.e. improves the coverage performance by the network. Mathematically, we would like to solve the following optimization problem

$$\begin{aligned} &\underset{u_i, v_i}{\text{maximize}} && F \\ &\text{subject to} && I_i > 0, \forall i. \end{aligned} \tag{3.5}$$

3.2 Performance-guaranteed Coverage Algorithm

In this subsection, a new distributed algorithm is proposed combining the proposed power-aware coverage algorithm and leader-following algorithm in order to guarantee that no isolated sensors exist at the final configuration. Since the isolated sensors have relatively no information, i.e. do not sense any events, they can not participate in the coverage task. Furthermore, random movements by the isolated sensors can not guarantee their participation in the coverage task and would result in a large amount of energy consumption which is not desired. Thus, they need to be guided to move into the region of interest to perform the coverage task. However, no external supervisor is allowed in order to keep the algorithm to be distributed. In this article, as a strategy, first a virtual supervisor called as leader is assigned between the sensors. The rest of the sensors then act as followers and will follow the leader(s) based on the leader-following algorithm until they gain sufficient information. It is assumed that the sensors could receive the position and orientation of its neighbors and also information sensed by its neighbors by using the communication network. Moreover, the communication graph between the sensors is given by

Assumption 2. *The communication graph is static and connected.*

Note that this assumption leads to unlimited communication radius, i.e. $r \to \infty$ which results in a fully connected graph.

3.2.1 Leader election

At the initial deployment, the sensors elect leader(s) among themselves which is defined as follows.

Definition 3.2. Leader sensor(s) l are sensors located within region of interest and have relatively more information. Mathematically,

$$l = \underset{i}{\operatorname{argmax}}\ I_i, \tag{3.6}$$

in the case of a single leader, or

$$\text{leaders } l \text{ are sensors with } I_l(q) > \mu, \tag{3.7}$$

in the case of multiple leaders, where $\mu = \frac{1}{N}\sum_{i=1}^{N} I_i(q)$ which is the average of information sensed by all sensors.

The leader is elected in a distributed manner using the combination of a voting algorithm and a simple implementation of broadcast algorithm called *flooding*, see Algorithm 1 for the detail. In addition, let us denote the set of leaders by N^l.

Remark 3.2. The election of multiple leaders is advantageous to increase the robustness of the algorithm in the presence of sensor failures, i.e. if one of the leader sensors fail, the algorithm could still be executed.

Algorithm 1 Leaders voting algorithm for multi-leaders case

Require: N

  $i \Leftarrow 1$; $L_d \Leftarrow 0$; $n \Leftarrow 1$; $I_t \Leftarrow 0$ (Initialization)

Require: $I_i(q)$

  $Store[i] \Leftarrow I_i$; $I_t \Leftarrow I_i$,

Ensure: : $\mu = \gamma \frac{I_t}{n}$,

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for j = 1 to N, j ≠ i, do
  Sent Store[i] to Agent j
  Store[j] ⇐ I_j; n ⇐ n + 1; I_t ⇐ I_t + I_j
  (Leader Assignment)
  if n = N then
    if μ ≤ Store[i] then
      L_d ⇐ L_d + 1; StoreL[L_d][i] ⇐ Store[i]
    end if
    if μ ≤ Store[j] then
      L_d ⇐ L_d + 1; StoreL[L_d][j] ⇐ Store[j]
    end if
    Go to END
  else
    for k = 1 to N, k ≠ i, k ≠ j, do
      Sent Store[i] and Store[j] to Agent k
      Store[k] ⇐ I_k; n ⇐ n + 1; I_t ⇐ I_t + I_k
      (Leader Assignment)
      if n = N then
        if μ ≤ Store[i] then
          L_d ⇐ L_d + 1; StoreL[L_d][i] ⇐ Store[i]
        end if
        if μ ≤ Store[j] then
          L_d ⇐ L_d + 1; StoreL[L_d][j] ⇐ Store[j]
        end if
        if μ ≤ Store[k] then
          L_d ⇐ L_d + 1; StoreL[L_d][k] ⇐ Store[k]
        end if
        Go to END
      else
        .
        .
      end if
    end for
  end if
end for
END :
```

3.2.2 Leader-Following Algorithm

Next we review the leader-following algorithm. Since the graph is fully connected, each sensor can communicate with the rest of the sensors. In other words, each sensor has at least one leader. The sensor which is not a leader is then following the leader using a leader-following algorithm defined as follows

$$\begin{aligned} u_i^{\mathsf{lf}} &= l_f \sum_{l \in N^l} (s_l - s_i), i \notin N^l & (3.8) \\ v_i^{\mathsf{lf}} &= h_f \sum_{l \in N^l} (\theta_l - \theta_i), i \notin N^l & (3.9) \end{aligned}$$

where $l_f, h_f > 0$ are the controller gains. Under assumption 2, the followers will approach either the leader or the convex hull spanned by the leaders when the leaders move in a slow speed (Ren, Beard and Atkins, 2007).

We are now ready to present a new control law which guarantees that no isolated sensors exist in the final configuration of the mobile sensors. First we introduce the following assumption.

Assumption 3. *There exists at least one non-isolated sensor, i.e.,* $\exists i, I_i \neq 0$.

The proposed distributed coverage control laws are then given as follows.

$$\begin{aligned} u_i &= u_i^{\mathsf{ce}} + \alpha_i u_i^{\mathsf{lf}} \\ v_i &= v_i^{\mathsf{ce}} + \alpha_i v_i^{\mathsf{lf}}, \end{aligned} \qquad (3.10)$$

where $\alpha_i(I_i) = 0$ if $I_i > \gamma_i I_l$ for a single leader case or $I_i \geq \gamma_i \mu$ for multiple leaders case and $\alpha_i(I_i) = 1$ otherwise where $0 < \gamma_i \leq 1$ is a tuning parameter. Physically, it means that when a sensor has gathered sufficient information, then it will stop following the leader and only performs the coverage. Thus, by implementing the control law (3.10), the participation of all sensors in the coverage task is guaranteed as given by the following Theorem.

Theorem 3.2. *Consider robotic visual sensors whose kinematics given by (2.1) and the control input is given by (3.10). Thus, under assumption 1-3, in the final configuration there will be no isolated sensors, i.e. all sensors participate in the coverage task.*

Proof. From assumption 3, there will always be at least a leader in the space. Moreover, from Assumption 2 since the graph is connected, then by implementing the leader-following algorithm (3.8), (3.9), all the followers will approach the leader or the convex hull spanned by the leaders until they have a sufficient information, i.e. $I_f > \gamma_f I_l > 0$ or $I_f \geq \gamma_f \mu > 0$. Thus, it is guaranteed that in the end of the deployment, there are no isolated sensors, i.e. $I_i > 0, \forall i$. □

Remark 3.3. Another advantage of the proposed algorithm is that the convergence speed is improved. From (3.10), the speed of the followers are the sum of the control input of the coverage algorithm (3.1), (3.2) and the leader-following algorithm (3.8), (3.9). Thus (3.8), (3.9) accelerates the movement of the followers, i.e. the convergence speed of (3.10) is faster than (3.1), (3.2).

Remark 3.4. The condition for switching into a pure coverage algorithm, i.e. $\alpha_i = 0$ in the case of multiple leaders is not unique. Another possibility is when $I_i \geq \gamma_i \mu_l$ where $\mu_l = \frac{1}{|N^l|} \sum_{j=1}^{|N^l|} I_j, j \in N^l$ and $|N^l|$ is the cardinality of N^l or when $I_i > \gamma_i I_{l^*}$, $l^* = \mathrm{argmax}_{l \in N^l} \; I_l$.

Formally, the proposed algorithm can be written as Algorithm 2.

Algorithm 2 Proposed distributed algorithm for sensor i ($i \in \{1, \cdots, N\}$)

Require: $I_i, \cdots, I_N$

loop

Leader election is conducted among all sensors

if i is a leader or $I_i > \gamma_i I_l$ ($I_i \geq \gamma_i \mu$ for multiple leaders case) **then**

$\alpha_i \Leftarrow 0$ (autonomous deployment mode)

$u_i = u_i^{\text{ce}}$

else

$\alpha_i \Leftarrow 1$ (following mode)

$u_i = {u_i}^{ce} + {u_i}^{\text{lf}}$

end if

end loop

4 Simulations

In this section, the proposed algorithms are evaluated through numerical simulations. The environment is a rectangle of size 400×600 (meter). It is assumed that there are 4 mobile sensors with sensing radius R = 100 meter and initial energy storage is equal to maximum energy storage $E_{\max} = 800$. The density function is $\phi(q) = 18 - 0.1||q - p_t||$ with $p_t = [400, 600]$. The sensing performance of the sensor is given by (2.4). Furthermore, it is assumed that sensor 1 is isolated from the region of interest, i.e. $I_1 = 0$ where its position is equal to $s_1 = [80, 30]$ and its orientation is equal to $\theta_1 = 90°$ while the rest are located at $s_2 = [80, 80]$ and $\theta_2 = 0°$; $s_3 = [80, 130]$ and $\theta_3 = 45°$; $s_4 = [80, 180]$ and $\theta_4 = 0°$. First, the standard coverage algorithm in (Li and Cassandras, 2005) is applied. As can be seen from Fig. 2(a), at the end of the deployment, sensor 1 does not participate in the coverage since it does not sense anything at its initial position which results in a degradation of the objective function as shown in Fig. 4. In addition, the degradation of the objective function is also caused by the decreasing of the sensing range of the sensors since the control law do not take into account the energy consumption of the sensors. Next we apply the proposed control law with multiple leaders by letting sensors which do not sense any information or sense very few information to follow the sensor with some information, called leader and taking into account the energy consumption of the sensors in their movements. First leader(s) defined in (3.7) is elected among the sensors. At the initial deployment sensor 3 and sensor 4 which have information value above the average are selected as the leaders. After 550 steps, sensor 2 is added to the leader list. The rest of the sensors, including the isolated sensor then follow the leaders until their information reach 87.5$\%$ (we choose $\gamma_i = 0.875, \forall i$) of the average information value as shown in Fig. 6. After 940 steps, all sensors execute autonomous deployment mode, i.e. their information values exceed the average of the sensors'. The trajectories and the evolution of orientation of the mobile sensors are depicted in Fig. 2 and Fig. 3 respectively. As can be observed from Figs. 2(c), 4, 5, all sensors participate in the coverage task which is indicated by the improvement of the objective function value. Furthermore, the energy consumption is less than the standard algorithm. Note that when we apply the proposed algorithm without

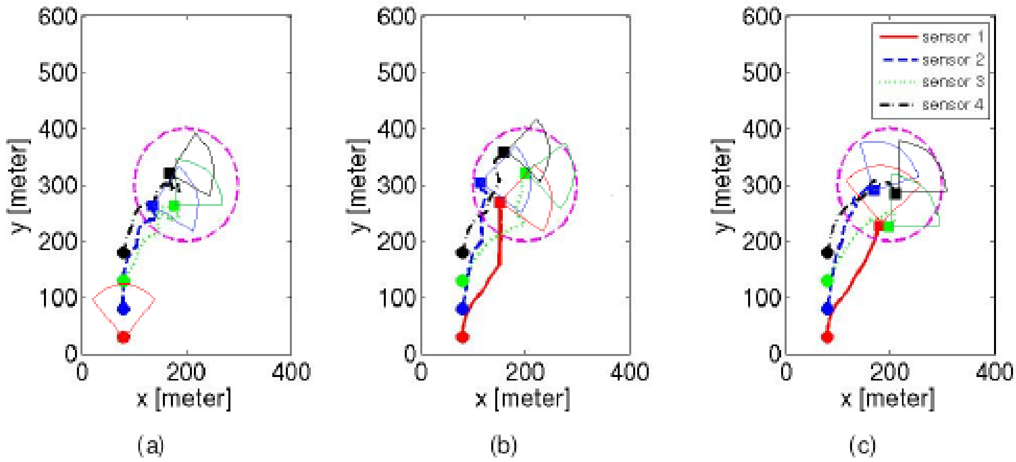

Figure 2: The trajectories and final configuration of robotic visual sensors with (a) standard coverage algorithm; (b) proposed coverage algorithm without considering energy consumption; (c) proposed coverage algorithm with energy consideration. The circles and squares represent the initial and final position of the sensors respectively.The area inside the dashed circle has higher density function

considering the energy consumption of the sensors, even though at the final deployment there exists no isolated sensors and the objective function is improved as shown in Fig. 2(b) and Fig. 4 respectively, the sensors consume more energy as can be seen from Fig. 5.

5 Conclusions

In this article, we propose a novel distributed control law that guarantees the participation of all sensors in the network while at the same time maintains the energy utilization optimally. In order to guarantee the participation of all sensors, the standard coverage algorithm is combined with the leader-following algorithm. The idea is that the sensors which have more information guide the sensors with no information on the environment until they gain sufficient information. Furthermore, the standard coverage algorithm is modified in order to take into account the energy consumption of the sensors by considering the current power state of the sensor in the model The effectiveness of the proposed control law is validated by numerical simulations.

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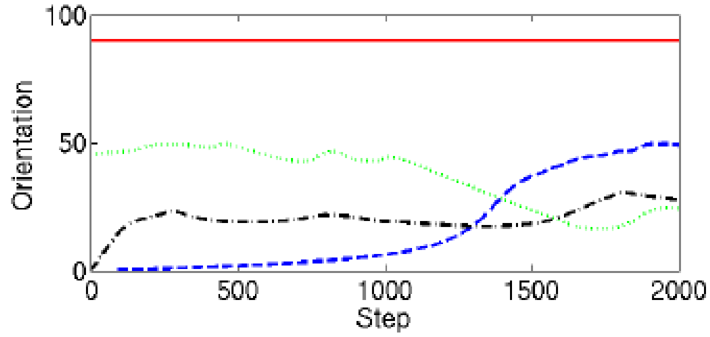
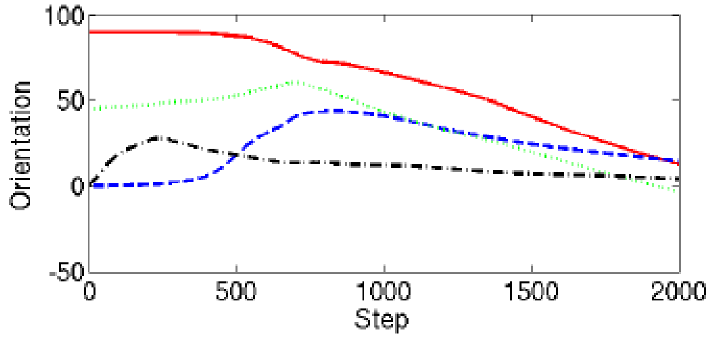
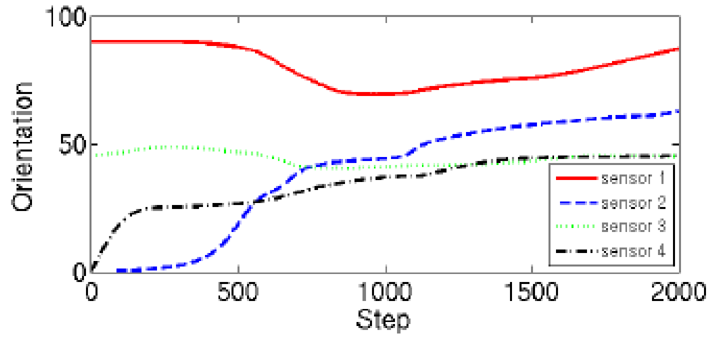


(a) Standard algorithm without considering energy consumption



(b) Proposed algorithm without considering energy consumption



(c) Proposed algorithm with energy consideration

Figure 3: Comparison of the evolution of orientation of the robotic visual sensors.

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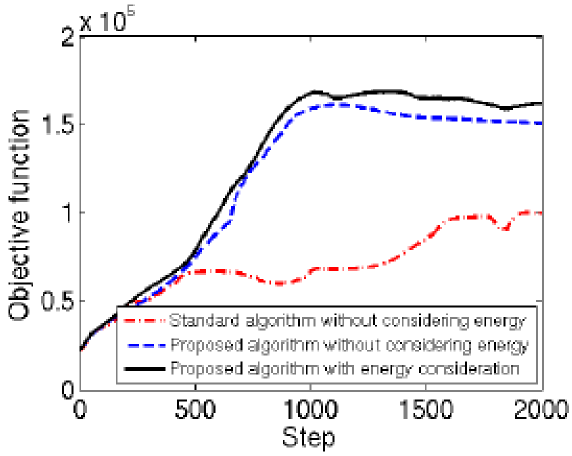

Figure 4: The objective function using the standard algorithm, the proposed algorithm without considering energy consumption and the proposed algorithm with energy consideration. As can be seen, the proposed algorithm with energy consideration results in an improved coverage performance indicated by the higher value of the objective function.

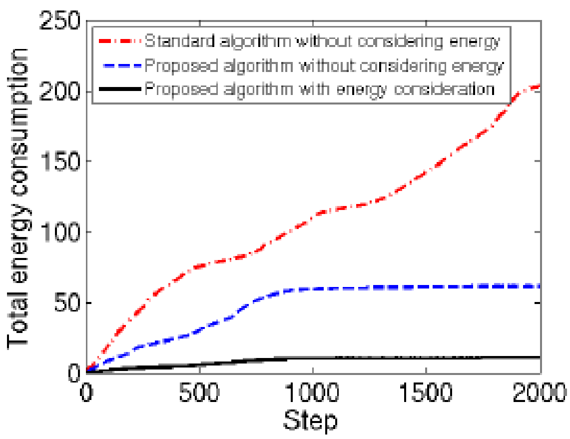

Figure 5: Comparison of energy consumption of the robotic visual sensors network.

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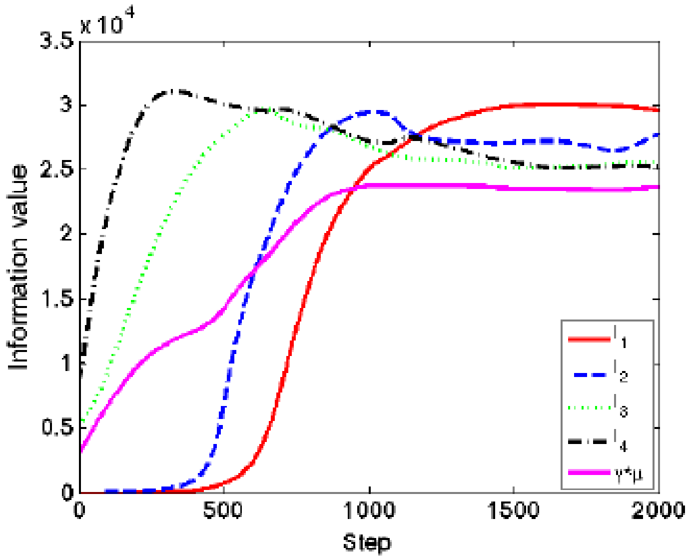

Figure 6: Information value of the robotic visual sensors. The isolated sensors and sensors with less information value follow the leader until their information value exceed 87.5% of average information value.

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